

SECTION I, MECHANICS

1. E

Let y denote the total distance that the rock falls and let T denote the total time of the fall.

Then $\frac{1}{2}y = \frac{1}{2}gt_1^2$ and $y = \frac{1}{2}gT^2$, so

$$\frac{t_2}{t_1} = \frac{T - t_1}{t_1} = \frac{T}{t_1} - 1 = \frac{\sqrt{2y/g}}{\sqrt{y/g}} - 1 = \sqrt{2} - 1$$

2. E

A trick for solving problems of this nature is to make up numbers for a and t . Pretend that total time = 10 seconds, and acceleration = 2 m/s². Then make a table:

Time, s	Speed, m/s
0	0
1	2
9	18
10	20

So in the first second, $v_{\text{avg}} = 1$ m/s, and $d = 1$ m. In the last (tenth) second, $v_{\text{avg}} = 19$ m/s, and $d = 19$ m. The answer to the question is $19 \text{ m} - 1 \text{ m} = 18 \text{ m}$. Which answer choice gives us 18 m?

(A) $a(t + 1 \text{ s})^2 = (2 \text{ m/s}^2)(10 \text{ s} + 1 \text{ s})^2 = (2 \text{ m/s}^2)(11 \text{ s})^2 = (2 \text{ m/s}^2)(121 \text{ s}^2) = 242 \text{ m}$

(B) $a(t - 1 \text{ s})^2 = (2 \text{ m/s}^2)(10 \text{ s} - 1 \text{ s})^2 = (2 \text{ m/s}^2)(9 \text{ s})^2 = (2 \text{ m/s}^2)(81 \text{ s}^2) = 162 \text{ m}$

(C) $at^2 = (2 \text{ m/s}^2)(10 \text{ s})^2 = (2 \text{ m/s}^2)(100 \text{ s}^2) = 200 \text{ m}$

(D) $a(t + 1 \text{ s})(1 \text{ s}) = (2 \text{ m/s}^2)(10 \text{ s} + 1 \text{ s})(1 \text{ s}) = (2 \text{ m/s}^2)(11 \text{ s})(1 \text{ s}) = 22 \text{ m}$

(E) $a(t - 1 \text{ s})(1 \text{ s}) = (2 \text{ m/s}^2)(10 \text{ s} - 1 \text{ s})(1 \text{ s}) = (2 \text{ m/s}^2)(9 \text{ s})(1 \text{ s}) = 18 \text{ m}$

Therefore (E) is the correct answer.

If you really want to do it algebraically, here's one way. Solve for the distance traveled in the first second.

$$\begin{aligned}d &= v_0 \Delta t + \frac{1}{2} a \Delta t^2 \\&= (0 \text{ m/s})(1 \text{ s}) + (1/2)(a)(1 \text{ s})^2 \\&= (1/2)(1 \text{ s}^2)a\end{aligned}$$

Next, solve for the distance in the final second.

$$\begin{aligned}
 d &= v_0 \Delta t + \frac{1}{2} a \Delta t^2 \\
 &= a(t - 1 \text{ s})(1 \text{ s}) + \frac{1}{2} (a)(1 \text{ s})^2 \\
 &= a(t - 1 \text{ s})(1 \text{ s}) + \frac{1}{2} (1 \text{ s}^2) (a)
 \end{aligned}$$

The difference between the two is $a(t - 1 \text{ s})(1 \text{ s})$ meters.

3. **A**

Integrating $a(t)$ gives $v(t)$:

$$v(t) = \int a(t) dt = \int 2t dt = t^2 + v_0$$

Since $v_0 = 0$, we have $v(t) = t^2$, which is never negative. This means the object never moves backward, so displacement is equal to the distance traveled. Now integrating $v(t)$ from $t = 0$ to $t = 3 \text{ s}$ gives the object's displacement during this time interval:

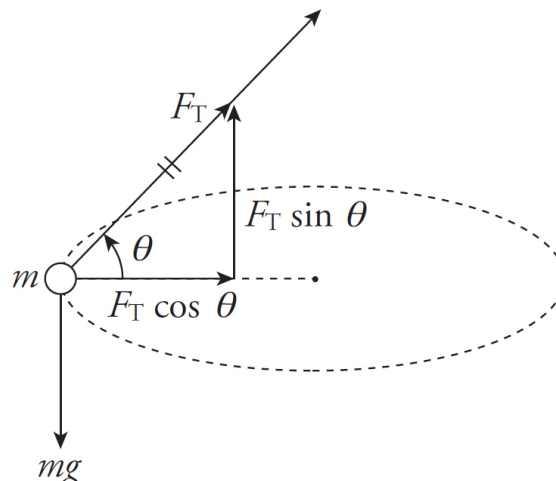
$$s = \int_0^3 v(t) dt = \int_0^3 t^2 dt = \frac{1}{3} t^3 \Big|_0^3 = 9 \text{ m}$$

4. **A**

Raising an object in ideal conditions involves overcoming gravity, which is a conservative force. Therefore, the work required will always be the same. If we think of work as $W = Fd \cos \theta$ and treat W as a constant, the purpose of an inclined ramp is to increase the distance of the movement so that the minimum force can be reduced.

5. **C**

The figure below shows that $F_T \sin \theta = mg$ and $F_T \cos \theta = mv^2/R = m\omega^2 R$:



We can eliminate θ from these equations by remembering that $\sin^2 \theta + \cos^2 \theta$ is always equal to 1:

$$\left(\frac{mg}{F_T} \right)^2 + \left(\frac{m\omega^2 R}{F_T} \right)^2 = 1 \Rightarrow \frac{m^2 g^2 + m^2 \omega^4 R^2}{F_T^2} = 1 \Rightarrow F_T = m\sqrt{\omega^4 R^2 + g^2}$$

That's enough information to get you to the correct answer, (C), but here's another way to think about this problem.



If you look at the image of $F_T \sin \theta$ and $F_T \cos \theta$, you might recognize that this is a right triangle, to which you can apply the Pythagorean theorem. $F_{T,x} = \frac{mv^2}{R} = m\omega^2 R$

and $F_{T,y} = mg$, which means that $F_T^2 = (mg)^2 + (m\omega^2 R)^2 \rightarrow F_T^2 = m^2(g^2 + \omega^4 R^2) \rightarrow m\sqrt{g^2 + \omega^4 R^2}$.

6. **A**

You can eliminate some choices right off the bat if you do some basic dimensional analysis. Choices (A), (B), and (C) all have dimensions of $\sqrt{GM/R}$. Looking up the units of G , you'll see that GM/R has units of $\text{N}\cdot\text{m}/\text{kg} = \text{m}^2/\text{s}^2$. Choices (D) and (E) have an extra factor of mass in the numerator ($\sqrt{\text{kg}}$) which doesn't belong in an expression of speed, so they can be ruled out. Because the initial height of the object is comparable to the radius of the Moon, we cannot simply use mgh for its initial potential energy. Instead, we must use the more general expression $U = -GMm/r$, where r is the distance from the center of the Moon. Notice that since $h = R$, the object's initial distance from the Moon's center is $h + R = R + R = 2R$. Conservation of Energy then gives

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 - \frac{GMm}{2R} &= \frac{1}{2}mv^2 - \frac{GMm}{R} \\ \frac{GMm}{2R} &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{GM}{R}} \end{aligned}$$

7. **A**

The formulas for frequency and period of a pendulum show that these values depend only on the length of the string and the acceleration of gravity. Because these are the same for both pendulums, (B), (C), and (D) can be eliminated. If the total mechanical energies of the systems are the same, that means the maximum kinetic energy, which is found at the bottom of the pendulum's movement, will be the same for both systems. Therefore,

$$\begin{aligned} KE_1 &= KE_2 \\ \frac{1}{2}mv_1^2 &= \frac{1}{2}(2m)v_2^2 \\ v_1 &= \sqrt{2}v_2 \end{aligned}$$

Thus, the system with the smaller mass will have the greater maximum speed. Through Process of Elimination, you can arrive at (A) without doing any further work; however, it's also helpful to know that the maximum angle of displacement must also be greater for the smaller mass. That is the only way to ensure the pendulum with the smaller mass will also have the same maximum potential energy.

8. **D**

Let F_T be the tension in each string. Then $F_{\text{net}} = ma$ becomes $mg - 2F_T = ma$. Also, the total torque exerted by the tension forces on the cylinder is $2rF_T$, so $\tau_{\text{net}} = I\alpha$ becomes $2rF_T = (\frac{1}{2}mr^2)\alpha$. Because the cylinder doesn't slip, $\alpha = a/r$, so $2F_T = \frac{1}{2}ma$. These equations can be combined to give $mg - \frac{1}{2}ma = ma$, which implies $mg = \frac{3}{2}ma$. Therefore, $a = \frac{2}{3}g$.

9. **B**

$$P = Fv = mgv$$

$$m = P/(gv)$$

$$= (1.5 \times 10^8 \text{ W})/[(10 \text{ m/s}^2)(150 \text{ m/s})] = 1.0 \times 10^5 \text{ kg}$$

10. **B**

The cylinder slides across the surface with acceleration $a = \mathbf{F}/m$ until time $t = T$, when it drops to zero (because $\mathbf{F}$ becomes zero). Therefore, from time $t = 0$ to $t = T$, the velocity is steadily increasing (because the acceleration is a positive constant), but, at $t = T$, the velocity remains constant. This is illustrated in graph (B).

11. **A**

Let m and M denote the mass of the satellite and Earth, respectively. The work required to lift the satellite to height h above the surface of Earth is

$$W = \Delta U = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R} \right) = \frac{GMmh}{R(R+h)}$$

Earth's gravitational pull provides the necessary centripetal force on the satellite, so $F_G = F_c$.

$$\frac{GMm}{(R+h)^2} = \frac{mv^2}{R+h} \Rightarrow mv^2 = \frac{GMm}{R+h} \Rightarrow K = \frac{GMm}{2(R+h)}$$

Because we're told that $W = K$, we find that

$$\frac{GMmh}{R(R+h)} = \frac{GMm}{2(R+h)} \Rightarrow \frac{h}{R} = \frac{1}{2} \Rightarrow h = \frac{1}{2}R$$

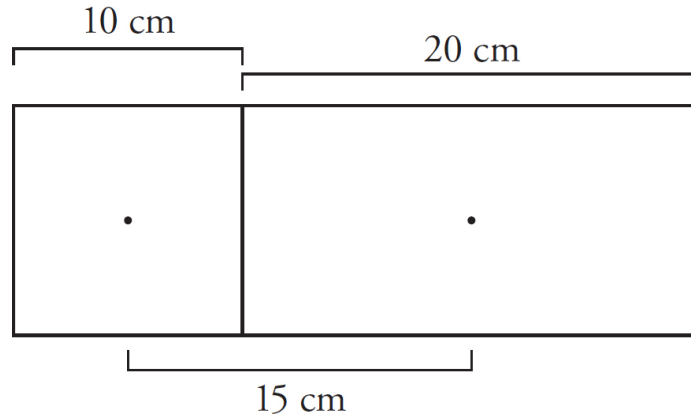
12. **E**

First, calculate the location of the collision. Given that the objects start 1 m apart and they move toward one another at a net rate of 10 m/s, 0.1 s will pass before the collision occurs. Therefore, the collision point will be 20 cm from the first object's starting point.

Next, calculate the velocity of the combined blocks post-collision using conservation of linear momentum for a perfectly inelastic collision:

$$\begin{aligned}
m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 &= (m_1 + m_2) \mathbf{v}_f \rightarrow \mathbf{v}_f \\
&= \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} \\
&= \frac{(10 \text{ kg})(2 \text{ m/s}) + (2 \text{ kg})(-8 \text{ m/s})}{10 \text{ kg} + 2 \text{ kg}} \\
&= \frac{1}{3} \text{ m/s}
\end{aligned}$$

Next, find the center of mass of the newly combined objects.



Treating the center of the first object as a 0 point, we get

$$x_{\text{CoM}} = m_1 x_1 + m_2 x_2 / (m_1 + m_2) = (10 \text{ kg})(0 \text{ m}) + (2 \text{ kg})(0.15 \text{ m}) / (10 \text{ kg} + 2 \text{ kg}) = 0.025 \text{ m}$$

This means that, in addition to the 0.3 m needed to get from the collision point to back to the midpoint, there is an additional 0.025 m that must be traveled because the center of mass location is not precisely the same as the collision location. Therefore

$$v = d/t \rightarrow t = d/v = (0.325 \text{ m}) / (1/3 \text{ m/s}) = 0.975 \text{ s}$$

13. **C**

Because the ball rebounded to the same height from which it was dropped, its takeoff speed from the floor must be the same as the impact speed. Calling *up* the positive direction, the change in linear momentum of the ball is $p_f - p_i = mv - (-mv) = 2mv$, where $v = \sqrt{2gh}$ (this last equation comes from the equation $\frac{1}{2}mv^2 = mgh$). The impulse-momentum theorem, $J = \Delta p$, now gives

$$F_{\text{avg}} \Delta t = 2m\sqrt{2gh} \Rightarrow F_{\text{avg}} = \frac{2m\sqrt{2gh}}{\Delta t} = \frac{2(0.08 \text{ kg})\sqrt{2(10 \text{ m/s}^2)(3.2 \text{ m})}}{0.04 \text{ s}} = 32 \text{ N}$$

14. **D**

Using the Big Five equation $\Delta\theta = \omega_0 t + \frac{1}{2}at^2$, we find that

$$\Delta\theta = \frac{1}{2}(2 \text{ rad/s}^2)(4.0 \text{ s})^2 = 16 \text{ rad}$$

Therefore, $\Delta s = r\Delta\theta = (0.1 \text{ m})(16 \text{ rad}) = 1.6 \text{ m}$. If you're worried about memorizing the Big Five equations, here's another method. The angular acceleration is 2.0 rad/s^2 , which means it gets 2.0 rad/s faster every second. It starts from rest. After 1 s, $\omega = 2 \text{ rad/s}$. After 2 s, $\omega = 4 \text{ rad/s}$. After 3 s, $\omega = 6 \text{ rad/s}$. After 4.0 s, $\omega = 8.0 \text{ rad/s}$. That makes the average

angular velocity during these four seconds 4.0 rad/s (ω increased linearly from 0 s to 8 s). Since the disk averages 4.0 rad/s for 4 seconds, the angular distance traveled will be 16 rad, which brings you right back to $\Delta s = r\Delta\theta$.

One final alternative is to make a graph with ω on the vertical axis and t on the horizontal axis. The slope of an ω vs. t plot is $\alpha = 2.0 \text{ rad/s}^2$. The area under the graph from 0 to 4 is the angular distance traveled by the point in the first 4.0 seconds.

15. **C**

The object's initial velocity from the slide is horizontal, so $v_{0y} = 0$, which implies that $\Delta y = -\frac{1}{2}gt^2$. Since $\Delta y = -R$,

$$-R = -\frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2R}{g}}$$

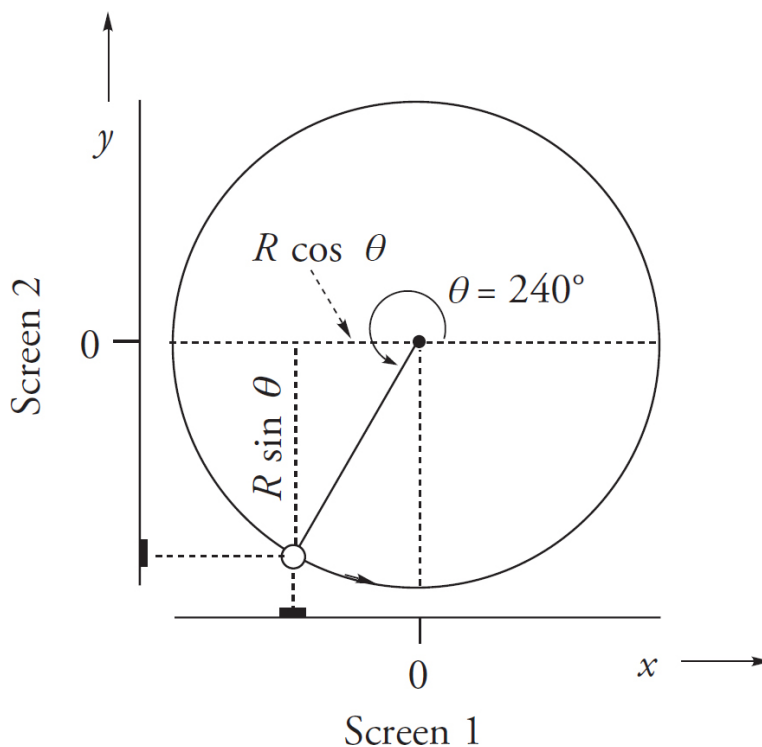
The (horizontal) speed with which the object leaves the slide is found from the conservation of energy: $mgR = \frac{1}{2}mv^2$, which gives $v = \sqrt{2gR}$. Therefore,

$$\Delta x = v_{0x}t = \sqrt{2gR} \cdot \sqrt{\frac{2R}{g}} = 2R$$

Since the object travels a horizontal distance of $2R$ from the end of the slide, the total horizontal distance from the object's starting point is $R + 2R = 3R$.

16. **A**

From the diagram below,



if $R \cos \theta = -(0.5)R$, then $\theta = 240^\circ$. Therefore, $y = R \sin \theta = R \sin 240^\circ = -(0.866)R$. Also, it is clear that the particle's subsequent motion will cause the shadow on Screen 2 to continue moving in the $-y$ direction.

17. **C**

The center of mass of the system is at a distance of

$$y_{cm} = \frac{(m)(0) + (2m)(L)}{m + 2m} = \frac{2}{3}L$$

below the mass m . With respect to this point, the clockwise torque produced by the force $\mathbf{F}$ has magnitude

$$\tau = rF = \left(\frac{2}{3}L - \frac{1}{4}L \right) F = \frac{5}{12}LF$$

Since the rotational inertia of the system about its center of mass is

$$I = \sum m_i r_i^2 = m \left(\frac{2}{3}L \right)^2 + (2m) \left(\frac{1}{3}L \right)^2 = \frac{2}{3}mL^2$$

the equation $\tau = I\alpha$ becomes

$$\frac{5}{12}LF = \left(\frac{2}{3}mL^2 \right) \alpha \Rightarrow \alpha = \frac{5}{8} \frac{F}{mL}$$

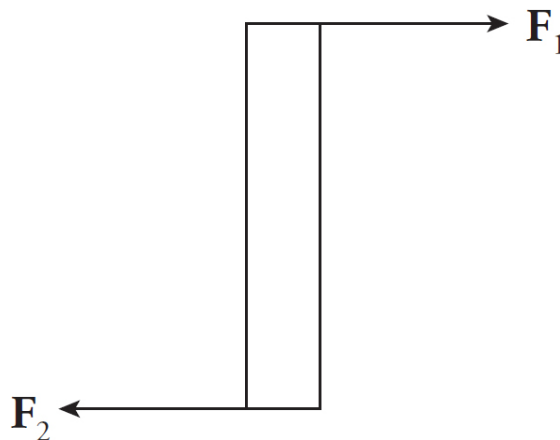
18. **E**

If the two vehicles have the same mass and speed before the collision and both come to a complete stop, then they have the same change in momentum. Impulse is just another name for change in momentum, so (B) is not true. If one vehicle is able to crumple, then it has more distance in which to come to a stop. Because it has more distance to work with, the process is drawn out over a greater time. Therefore, (D) is also not true, which means (E) is the correct answer.

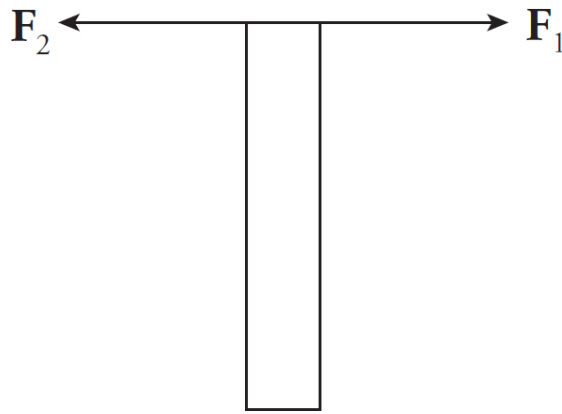
Both (A) and (C) are true statements.

19. **D**

Since $\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$, the bar cannot accelerate translationally, so (C) is false. The net torque does *not* need to be zero, as the following diagram shows, eliminating (A) and (B).



However, since $\mathbf{F}_2 = -\mathbf{F}_1$, (D) is true; one possible illustration of this is given below:



20. **B**

The value of g near the surface of a planet depends on the planet's mass and radius:

$$mg = \frac{GMm}{r^2} \Rightarrow g = \frac{GM}{r^2}$$

Therefore, calling this Planet X, we find that

$$g_X = \frac{GM_X}{r_X^2} = \frac{G(2M_{\text{Earth}})}{(2r_{\text{Earth}})^2} = \frac{1}{2} \frac{GM_{\text{Earth}}}{r_{\text{Earth}}^2} = \frac{1}{2} g_{\text{Earth}}$$

Since g is half as much on the planet as it is on Earth, the astronaut's weight (mg) will be half as much as well. The astronaut would weigh half of 800 N, which is 400 N, (B). Alternatively, you can plug in some numbers, although understand that this can introduce new errors if you're not careful with your calculations. This allows you to make some unreal assumptions with numbers that are easy to work with, in this case, that the Earth has a mass of 10 and a radius of 1. Now it's a lot easier to solve for Earth, $g = GM/R^2 = G \cdot 10/1^2 = 10G$. Planet X has twice the mass and radius of Earth, so for Planet X, $M = 20$ and $r = 2$. This gives $g = GM/R^2 = G \cdot 20/2^2 = 5G$. Since g is half as much on the planet as it is on Earth, the astronaut's weight will be half as much as well.

21. **C**

The work done by the force $F(x)$ is equal to the area under the given curve. The region under the graph can be broken into two triangles and a rectangle, and it is then easy to figure out that the total area is 50 N·m. Since work is equal to change in kinetic energy (and $v_i = 0$),

$$W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) \Rightarrow v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(50 \text{ N} \cdot \text{m})}{1.0 \text{ kg}}} = 10 \text{ m/s}$$

22. **C**

Since no external torques act on the star as it collapses (just like a skater when she pulls in her arms), angular momentum, $I\omega$, is conserved, and the star's rotational speed increases. The moment of inertia of a sphere of mass m and radius r is given by the equation $I = kmr^2$ (with $k = \frac{2}{5}$, but the actual value is irrelevant), so we have:

$$\omega_f = \frac{I_i}{I_f} \omega_i = \frac{kmr_i^2}{kmr_f^2} \omega_i = \frac{r_i^2}{r_f^2} \omega_i = \frac{r_i^2}{\left(\frac{1}{16}r_i\right)^2} \omega_i = 16^2 \omega_i$$

Therefore,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}I_f\omega^2}{\frac{1}{2}I_i\omega_i^2} = \frac{\frac{1}{2}kmr_f^2\omega^2}{\frac{1}{2}kmr_i^2\omega_i^2} = \frac{r_f^2\omega^2}{r_i^2\omega_i^2} = \frac{\left(\frac{1}{16}r_i\right)^2 \left((16^2\omega_i)\right)^2}{r_i^2\omega_i^2} = 16^2$$

23. **C**

The projectile's horizontal speed is $v_0 \cos 60^\circ = v_0 \cdot 1/2 = 20 \text{ m/s}$, so it reaches the wall in 1.0 s.

24. **B**

Start with the formula for electric force.

$$F_e = [1/(4\pi\epsilon_0)](q_1q_2/r^2)$$

$$\epsilon_0 = [1/(4\pi F_e)](q_1q_2/r^2)$$

Next, substitute the terms with the proper units.

$$\begin{aligned} [\epsilon_0] &= (1/\text{N})(\text{C}^2/\text{m}^2) \\ &= [1/(\text{kg m/s}^2)](\text{C}^2/\text{m}^2) = (\text{s}^2 \text{ C}^2)/(\text{kg m}^3) \end{aligned}$$

Last, just substitute the letters as the problem describes, and you end up with (B).

25. **D**

Right off the bat, you can eliminate (A) and (E), as they are equivalent to one another. Whether you noticed that or not, the diagram given with the question shows that after the clay balls collide, they move in the $-y$ direction, which means that the horizontal components of their linear momenta canceled. In other words, $p_{1x} + p_{2y} = 0$:

$$\begin{aligned} m_1 v_{1x} + m_2 v_{2x} &= 0 \\ (m_1 v_1 \sin 45^\circ) + (m_2 v_2 \sin 60^\circ) &= 0 \\ v_2 &= \frac{m_1 v_1 \sin 45^\circ}{m_2 \sin 60^\circ} \\ &= \frac{2 \sin 45^\circ}{\sin 60^\circ} v_1 \end{aligned}$$

26. **B**

The horizontal position of the blocks can be given by the equation

$$x = A \cos(\omega t + \phi_0)$$

where A is the amplitude, ω is the angular frequency, and ϕ_0 is the initial phase. Differentiating this twice gives the acceleration:

$$a = \frac{d^2 x}{dt^2} = -A\omega^2 \cos(\omega t + \phi_0) \Rightarrow a_{\max} = A\omega^2$$

This means that the maximum force on block m is $F_{\max} = ma_{\max} = mA\omega^2$. The static friction force must be able to provide this same amount of force; otherwise, the block will slip. Therefore,

$$\begin{aligned} F_{\text{static, max}} &\geq mA\omega^2 \\ \mu mg &\geq mA\omega^2 \\ A &\leq \frac{\mu mg}{m\omega^2} = \frac{\mu g}{(2\pi f)^2} = \frac{(0.80)(10 \text{ m/s}^2)}{4\pi^2(2.0 \text{ Hz})^2} \approx 0.05 \text{ m} = 5.0 \text{ cm} \end{aligned}$$

27. **D**

The ball strikes the floor with speed $v_1 = \sqrt{2gH_1}$, and it leaves the floor with speed $v_2 = \sqrt{2gH_2}$. Therefore,

$$e = \frac{v_2}{v_1} = \frac{\sqrt{2gH_2}}{\sqrt{2gH_1}} = \sqrt{\frac{H_2}{H_1}}$$

28. **D**

The center of rotation is the center of mass of the plate, which is at the geometric center of the square because the plate is homogeneous. Since the line of action of the force coincides with one of the sides of the square, the lever arm of the force, ℓ , is simply equal to $\frac{1}{2}s$. Therefore,

$$\tau = \ell F = \frac{1}{2}sF = \frac{1}{2}(0.40 \text{ m})(10 \text{ N}) = 2.0 \text{ N}\cdot\text{m}$$

29. **A**

Since gravity is a conservative force, the actual path taken is irrelevant. If the block rises a vertical distance of $z = 3 \text{ m}$, then the work done by gravity is

$$W = -mgz = -(2.0 \text{ kg})(10 \text{ N/kg})(3 \text{ m}) = -60 \text{ J}$$

30. **C**

Applying conservation of linear momentum for a perfectly inelastic equation, we see that the speed of the combined objects just after the collision will be

$$\begin{aligned} m_1\mathbf{v}_1 + m_2\mathbf{v}_2 &= (m_1 + m_2)\mathbf{v}_f \rightarrow \mathbf{v}_f \\ &= \frac{m\mathbf{v} + 0}{M + m} \end{aligned}$$

This speed means the block has additional kinetic energy, which will be converted into potential energy as the spring compresses.

$$KE = PE$$

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}kx^2 \rightarrow x \\ &= v\sqrt{\frac{m}{k}} \\ &= \left(\frac{mv}{M+m}\right)\sqrt{\frac{M+m}{k}} \\ &= mv\sqrt{\frac{1}{k(M+m)}}\end{aligned}$$

31. **D**

Consider an infinitesimal slice of width dx at position x ; its mass is $dm = \lambda dx = kx dx$. Then, by definition of rotational inertia,

$$I = \int r^2 dm = \int_0^L x^2 (kx dx) = \frac{1}{4} kx^4 \Big|_0^L = \frac{1}{4} kL^4$$

Because the total mass of the rod is

$$M = \int dm = \int_0^L kx dx = \frac{1}{2} kx^2 \Big|_0^L = \frac{1}{2} kL^2$$

we see that $I = \frac{1}{2}ML^2$.

32. **E**

Points of equilibrium occur when dU/dx is equal to zero:

$$\frac{dU}{dx} = 0 \Rightarrow 3(x-2)^2 - 12 = 0 \Rightarrow (x-2)^2 = 4 \Rightarrow x = 0 \text{ or } x = 4$$

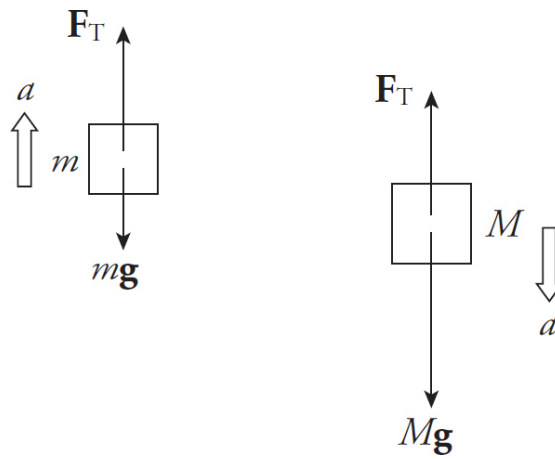
There's more than one way to determine which equilibrium points are stable equilibrium points. One way is to graph $U(x)$ and see which equilibrium point is a minimum. Another way is to use the first derivative. The following explanation uses the second derivative test, but feel free to use the method you're most comfortable with. Points of *stable* equilibrium occur when $dU/dx = 0$ and d^2U/dx^2 is positive (because then the equilibrium point is a relative minimum). Since

$$\frac{d^2U}{dx^2} = 6(x-2)$$

the point $x = 4$ is a point of stable equilibrium ($x = 0$ is unstable).

33. **D**

Since the pulley is not accelerating, it can serve as the origin of an inertial reference frame. Applying Newton's Second Law to the masses using the following free-body diagrams,



gives

$$F_T - mg = ma \text{ and } Mg - F_T = Ma$$

Adding these equations eliminates F_T , and we find that

$$Mg - mg = (m + M)a \Rightarrow a = \frac{M - m}{M + m}g$$

The acceleration of block m is $(M - m)g/(M + m)$ upward, and the acceleration of block M is this value, $(M - m)g/(M + m)$, downward, which is (D).

Alternatively, you can treat the two objects as one system. Tension is an internal force and cancels. The net force on the system is $Mg - mg$. The total mass is $M + m$. If you simplify this expression, you get the same answer, (D).

34. E

This can be solved using kinematics or work and energy. We'll use the latter solution because it is faster, but both methods are valid.

First, determine how much potential energy is converted to kinetic for each meter the block slides down the ramp.

$$\Delta PE = -\Delta KE \rightarrow mg\Delta h = -\Delta KE \rightarrow \Delta KE = -(mg(-d\sin\theta_r)) = -(2 \text{ kg})(10 \text{ m/s}^2)(-1 \text{ m})(0.45) = 9 \text{ J}$$

On a frictionless portion, this 9 J will convert to kinetic without issue. On the friction portions, however, some energy will be lost due to the friction. Each meter the block slides over the friction surface loses

$$W_{ff} = F_f d \cos\theta = (\mu F_N) d \cos\theta = \mu(mg \cos\theta_r) d \cos\theta = \left(\frac{2}{3}\right)(2 \text{ kg})(10 \text{ m/s}^2)(0.9)(1)(-1) = -12 \text{ J}$$

The $\cos\theta$ term is -1 because that θ is 180° , the angle between the force of friction and the movement of the block. It is not the angle of the ramp.

This means that each meter of sliding on a friction portion results in +9 J from the conversion of PE and -12 J from the friction, producing a net result of -3 J per meter when sliding over the friction portions of the ramp. The ramp's pattern of friction and non-friction will be

1 m without, 1 m with, 1 m without, 2 m with, 1 m without, 3 m with, 1 m without, 4 m with, 1 m without, 5 m with (total of 20 m)

This produces KE of

$$+9 \text{ J} - 3 \text{ J} + 9 \text{ J} - 6 \text{ J} + 9 \text{ J} - 9 \text{ J} + 9 \text{ J} - 12 \text{ J} + 9 \text{ J} - 15 \text{ J} = 0 \text{ J}$$

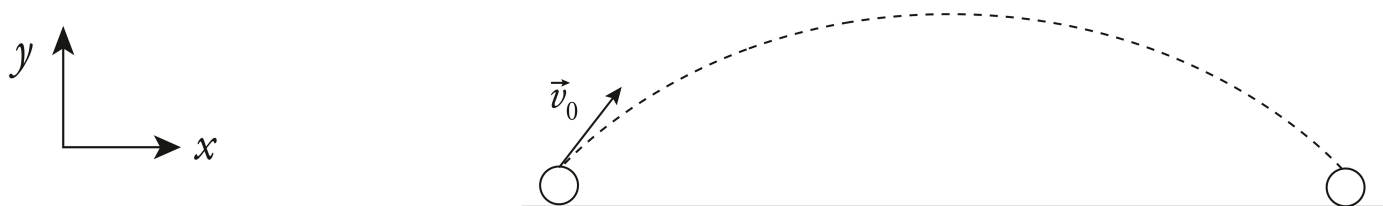
Although the block has no kinetic energy after passing the 5 m friction portion, it will continue because it is now on another frictionless portion. It gains the 9 J from this frictionless portion. This is enough to travel through 3 m of the next friction portion. Adding these additional 4 m to the previous 20 m gives the final answer of 24 m.

35. E

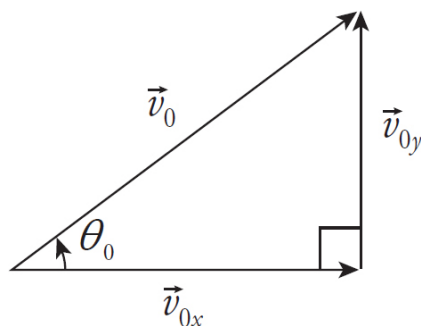
The maximum net force on the object occurs when all three forces act in the same direction, giving $F_{\text{net}} = 3F = 3(4 \text{ N}) = 12 \text{ N}$, and a resulting acceleration of $a = F_{\text{net}}/m = (12 \text{ N})/(2 \text{ kg}) = 6 \text{ m/s}^2$. These three forces could not give the object an acceleration greater than 6 m/s^2 . Because the question looks for the acceleration that is NOT possible, the answer is 8 m/s^2 , (E).

SECTION II, MECHANICS

1. This problem involves motion in two dimensions, so it's important to begin by drawing a large picture of the situation. Note that our coordinate system is set up in the usual way with x pointing to the right, y pointing up, and the origin at the point where the projectile is launched.



Next, draw a second diagram that splits $\vec{v}_0$ into vertical and horizontal components. Note that you can use trigonometric properties to derive the values of v_{0x} and v_{0y} .



Now we're ready to start answering the questions.

- (a) For this problem, time of flight is determined by vertical quantities. This is because the flight ends when the object hits the ground, that is, when the y -coordinate is zero again. If $\Delta y = 0$, the projectile is either beginning or ending its flight.

You need to find t and you're missing $v_{f,y}$, so use Big Five #3.

$$\Delta y = v_{0,y}t + \frac{1}{2}a_yt^2$$

$$0 = (v_0 \sin \theta)t + \frac{1}{2}(-g)t^2$$

$$\frac{1}{2}gt^2 - (v_0 \sin \theta)t = 0$$

$$t \left(\frac{g}{2}t - v_0 \sin \theta \right) = 0$$

At the beginning of the flight, t is equal to 0. However, at the end of the flight, you would have the following:

$$\frac{g}{2}t - v_0 \sin \theta = 0$$

$$\frac{g}{2}t = v_0 \sin \theta$$

$$t = \frac{2v_0 \sin \theta}{g}$$

The total amount of time in the air, then, is $\frac{2v_0 \sin \theta}{g} - 0$, or just $\frac{2v_0 \sin \theta}{g}$.

As an alternative, you could use Big Five #2 to find the amount of time in which the projectile moves upward and then multiply that number by 2 to get the total time.

- (b) As with the previous step, you are looking for a vertical quantity; because it's t that you no longer care about and Δy that you need, use Big Five #5.

$$v_{fy}^2 = v_{oy}^2 + 2a(\Delta y)$$

At the top of the trajectory, $v_{fy} = 0$, which means you can now solve for Δy .

$$0 = v_{oy}^2 + 2a\Delta y$$

$$-v_{oy}^2 = 2a\Delta y$$

$$\frac{-v_{oy}^2}{2a} = \Delta y$$

If you plug in the known values of v_{oy} and a , you get the following:

$$\Delta y = \frac{-(v_0 \sin \theta)^2}{2(-g)}$$

$$\Delta y = \frac{v_0^2 \sin^2 \theta}{2g}$$

Because $y_0 = 0$, that value of Δy is also the value of y_{max} .

- (c) The object starts and ends on the ground ($y = 0$), so its net vertical displacement is 0.
 (d) You're asked for a horizontal quantity, so let's review our horizontal components.

$$\Delta x = ?$$

$$v_x = v_0 \cos \theta$$

$$t = \frac{2v_0 \sin \theta}{g}$$

Velocity is defined as $v_x = \frac{\Delta x}{t}$, and this can be rewritten as $\Delta x = v_x t$. You've found two of these values already, so plug them in and solve.

$$\Delta x = (v_0 \cos \theta) \left(\frac{2v_0 \sin \theta}{g} \right)$$

$$\Delta x = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

(e) We're told that the range equals the maximum vertical displacement, so $\Delta x = y_{\max}$.

$$\frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin^2 \theta}{2g}$$

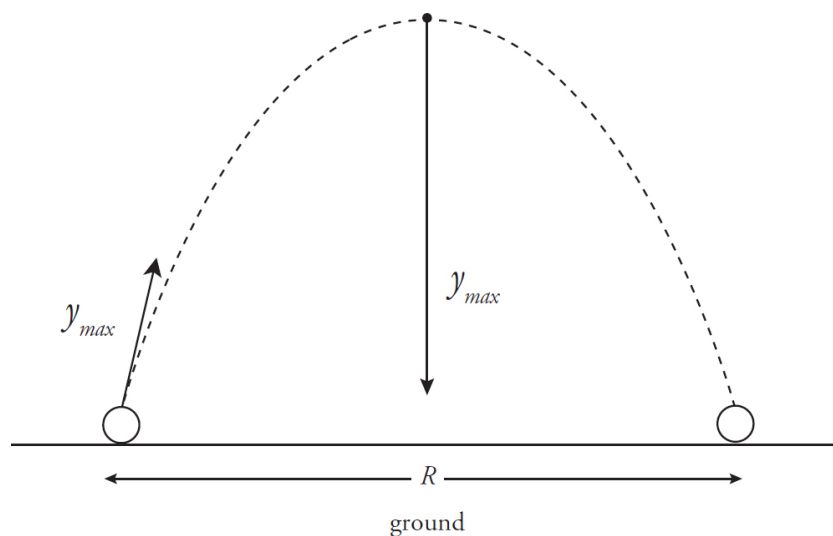
Cancelling out from this equation, you're left with the following:

$$2 \cos \theta = \frac{\sin \theta}{2}$$

$$4 = \tan \theta$$

$$\theta = \tan^{-1}(4)$$

You can check this by noting that the arctan of 4 is a fairly large angle (about 76°), which indicates that $\vec{v}_0$ points more upward than rightward.



2. (a) The mass contained in a sphere of radius $r < R$ is

$$M_{\text{within } r} = \rho V_{\text{within } r} = \frac{M}{\frac{4}{3}\pi R^3} \left(\frac{4}{3}\pi r^3 \right) = \frac{r^3}{R^3} M$$

so

$$F = -\frac{Gm}{r^2} \left(\frac{r^3}{R^3} M \right) = -\frac{GMm}{R^3} r$$

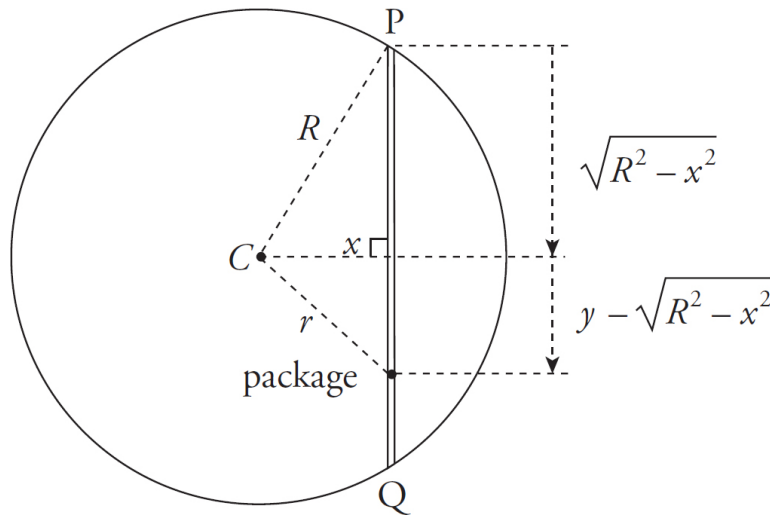
(b) Since $dU = -F(r) dr$,

$$\Delta U \Big|_R^r = \int_R^r dU = \int_R^r -F(r) dr = \int_R^r \frac{GMm}{R^3} r dr = \frac{GMm}{R^3} \left[\frac{1}{2} r^2 \right]_R^r = \frac{GMm}{2R^3} (r^2 - R^2)$$

(c) Conservation of Energy, $K_i + U_i = K_f + U_f$, can be written in the form $\Delta K = -\Delta U$, which gives

$$\begin{aligned} \frac{1}{2} mv^2 &= -\frac{GMm}{2R^3} (r^2 - R^2) \\ v &= \sqrt{\frac{GM}{R^3} (R^2 - r^2)} \end{aligned}$$

By applying the Pythagorean theorem to the two right triangles in the figure,



we see that

$$r^2 = x^2 + \left(y - \sqrt{R^2 - x^2} \right)^2$$

so the expression given above for v can be rewritten in the form

$$\begin{aligned} v &= \sqrt{\frac{GM}{R^3} \left[R^2 - x^2 - \left(y - \sqrt{R^2 - x^2} \right)^2 \right]} \\ &= \sqrt{\frac{GM}{R^3} \left[2y\sqrt{R^2 - x^2} - y^2 \right]} \end{aligned}$$

(d) (i) From the expression derived in part (c) above, we can see that the value of y which will maximize v is

$$y = \sqrt{R^2 - x^2}$$

This is the midpoint of the tunnel.

(ii) The maximum value for v is

$$v_{\max} = \sqrt{\frac{GM}{R^3} \left[R^2 - x^2 \right]}$$

3. (a) (i) Once the $3m$ puck sticks to the m puck, the center of mass of the system is

$$y_{\text{cm}} = \frac{(m+3m)(0) + (2m)(L)}{(m+3m) + 2m} = \frac{1}{3}L$$

above the bottom pucks on the rod.

- (ii) Applying Conservation of Linear Momentum, we find that

$$(3m)v = (3m + m + 2m)v'_{\text{cm}} \Rightarrow v'_{\text{cm}} = \frac{1}{2}v$$

- (iii) To keep this as straightforward as possible, choose our origin to be the center of mass of the system. Now, applying Conservation of Angular Momentum, we find that

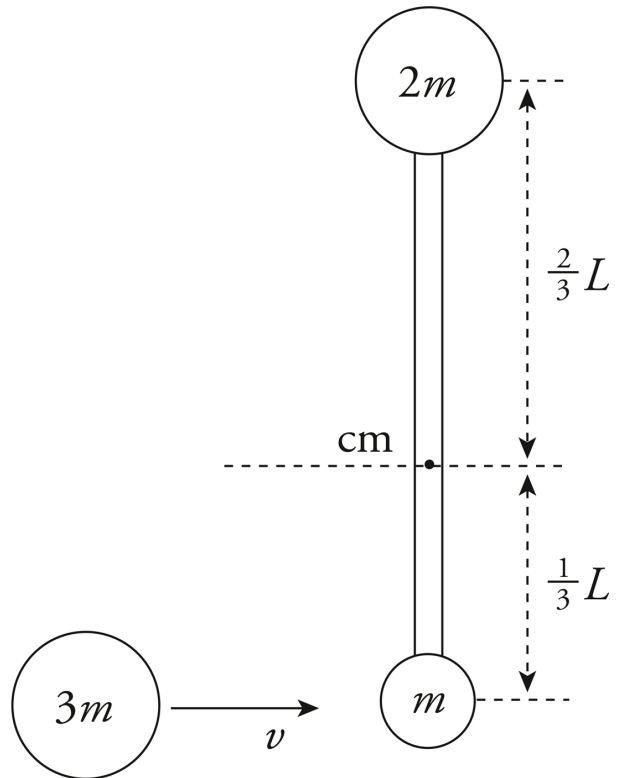
$$L_i = L_f$$

$$L_i = I_f \omega_f$$

$$\omega_f = \frac{L_i}{I_f}$$

$$= \frac{(\frac{1}{3}L)(3m)v}{\left[(m+3m)(\frac{1}{3}L)^2 + (2m)(\frac{2}{3}L)^2\right]}$$

$$= \frac{3v}{4L} \text{ (counterclockwise)}$$



- (b) The kinetic energy before the collision was the initial translational kinetic energy of the $3m$ puck:

$$K_i = \frac{1}{2}(3m)v^2$$

After the collision, the total kinetic energy is the sum of the system's translational kinetic energy and its rotational kinetic energy:

$$\begin{aligned} K' &= K'_{\text{translational}} + K'_{\text{rotational}} \\ &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}(3m + m + 2m)\left(\frac{1}{2}v\right)^2 + \frac{1}{2}\left[(3m + m)\left(\frac{1}{3}L\right)^2 + (2m)\left(\frac{2}{3}L\right)^2\right]\left(\frac{3v}{4L}\right)^2 \\ &= \frac{9}{8}mv^2 \end{aligned}$$

Therefore, the fraction of the initial kinetic energy that is lost due to the collision is

$$\frac{K - K'}{K} = \frac{\frac{3}{2}mv^2 - \frac{9}{8}mv^2}{\frac{3}{2}mv^2} = \frac{1}{4}$$

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