

Multiplying Matrices

A Matrix is an array of numbers surrounded by brackets.

A Matrix has the dimensions of *rows x columns*. An example of a 2x3 Matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Scalar Multiplication of a Matrix:

When a Matrix is multiplied by a number (AKA a “constant” or a “scalar”), just distribute the number across the brackets.

$$2 \begin{bmatrix} 4 & 0 \\ \frac{1}{2} & -9 \end{bmatrix} = \begin{bmatrix} 2 \times 4 & 2 \times 0 \\ 2 \times \frac{1}{2} & 2 \times -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 1 & -18 \end{bmatrix}$$

Multiplying a Matrix by Another Matrix:

In order to multiply two matrices, the inner dimensions must match; that is, the number of columns of the 1st Matrix must be equal to the number of rows in the 2nd Matrix.

The resulting Matrix will be of the outer dimensions; that is, the number of rows of the 1st Matrix by the number of columns of the 2nd Matrix. For instance:

A 2 x 3 Matrix CAN be multiplied by a 3 x 5 Matrix, and the result will be a 2 x 5 Matrix.

A 3 x 5 Matrix CANNOT be multiplied by a 2 x 3 Matrix. (NOTE: the order matters!)

When a Matrix is multiplied by another Matrix, compute the “Dot Product” of the rows and columns by multiplying matching members.

For example, [2 x 3] x [3 x 2]:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix}$$

- 1) Multiply the 1st member of the 1st row by the 1st member of the 1st column (1 x 7)
- 2) Multiply the 2nd member of the 1st row by the 2nd member of the 1st column (3 x 8)
- 3) Multiply the 3rd member of the 1st row by the 3rd member of the 1st column (5 x 9)
- 4) Add these products together (7 + 24 + 45)
- 5) Write this sum (76) in top left corner of the resulting 2 x 2 Matrix (where the 1st row and 1st column overlap)

$$\begin{bmatrix} 76 & \\ & \end{bmatrix}$$

Continuing on, find the Dot Product of the 1st row and the 2nd column, writing the sum of the products in the top right corner:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 76 & 1 \times 10 + 3 \times 11 + 5 \times 12 \\ 100 & 136 \end{bmatrix} = \begin{bmatrix} 76 & 103 \\ 100 & 136 \end{bmatrix}$$

Now move on to the Dot Product of the 2nd row and the 1st column, writing the sum of the products in the bottom left corner:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 76 & 103 \\ 2 \times 7 + 4 \times 8 + 6 \times 9 & 100 \end{bmatrix} = \begin{bmatrix} 76 & 103 \\ 100 & 136 \end{bmatrix}$$

And finally, do the Dot Product of the 2nd row and the 2nd column, writing the sum of the products in the bottom right corner:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 76 & 103 \\ 100 & 2 \times 10 + 4 \times 11 + 6 \times 12 \end{bmatrix} = \begin{bmatrix} 76 & 103 \\ 100 & 136 \end{bmatrix}$$

Your Turn!

A) $\begin{bmatrix} 4 & 3 \\ -2 & 5 \\ -4 & 6 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ -3 & -1 \end{bmatrix} =$

B) $\begin{bmatrix} -6 & -3y \\ -6x & -2 \end{bmatrix} \cdot \begin{bmatrix} -2x & 0 \\ 4y & -3 \end{bmatrix} =$

C) An ice cream stall sells green tea, mocha, and strawberry ice cream. A small size costs \$0.75, a medium costs \$1.25, and a large costs \$1.50. One day, the number of ice creams sold was:

	Small	Medium	Large
Green Tea	10	4	6
Mocha	2	8	9
Strawberry	5	11	7

For this ice cream stall on this day, which flavor of ice cream brought in the most income? What was the total income?